Matrix Determinant – Laplace and Sarrus Method

N.B
some of matrix picture might be good since it was generated from https://www.symbolab.com/solver/matrix-vector-calculator , meanwhile some of them are my handwriting.

https://github.com/antoniusrobotsoft

It has been so long since my last time playing with matrix. The last time I used matrix is when I was study about input and output optimization using leontief matrix. Since it has been so long, I decided to write an interesting topic about matrix called determinant.

What is matrix determinant ? Here is a short description from wikipedia:

In linear algebra, the determinant is a value that can be computed from the elements of a square matrix. The determinant of a matrix A is denoted det, det A, or |A|

First of all, it's only possible to find determinant of a matrix when a matrix has the same number of columns and rows. In order to calculate determinant of a matrix, there are many methods. There are 2 methods which I used frequently. They are : laplace method and sarrus method.

The most simple way to calculate a matrix determinant is when the matrix consists of 2 rows and 2 columns only.

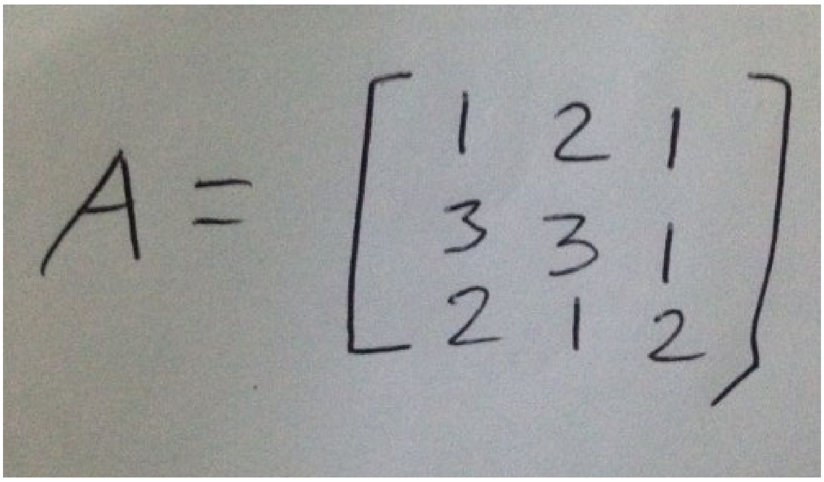
Example we have this matrix called U matrix :

$$\begin{pmatrix} 2 & 1 \\ 3 & 6 \end{pmatrix}$$

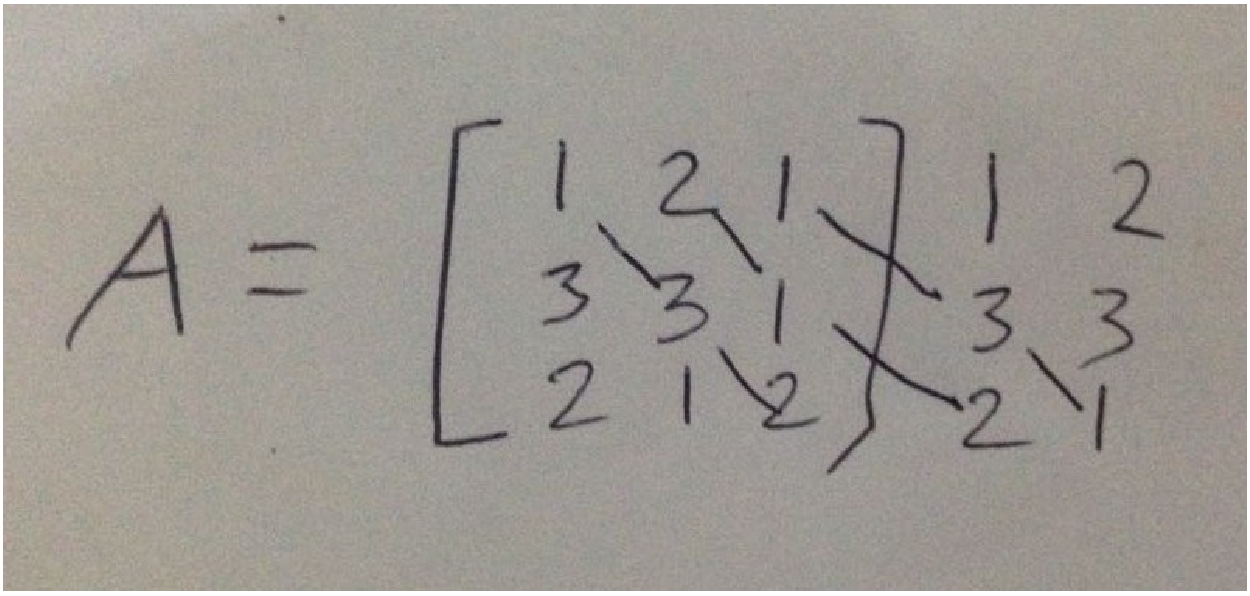
|U| = (2 * 6) – (1 * 3) = 9

Sarrus Method

Based on sarrus method : "3×3 matrix determinant" is the sum of the products of three diagonal north-west to south-east lines of matrix elements, minus the sum of the products of three diagonal south-west to north-east lines of elements. Suppose we have 3×3 matrix called A :



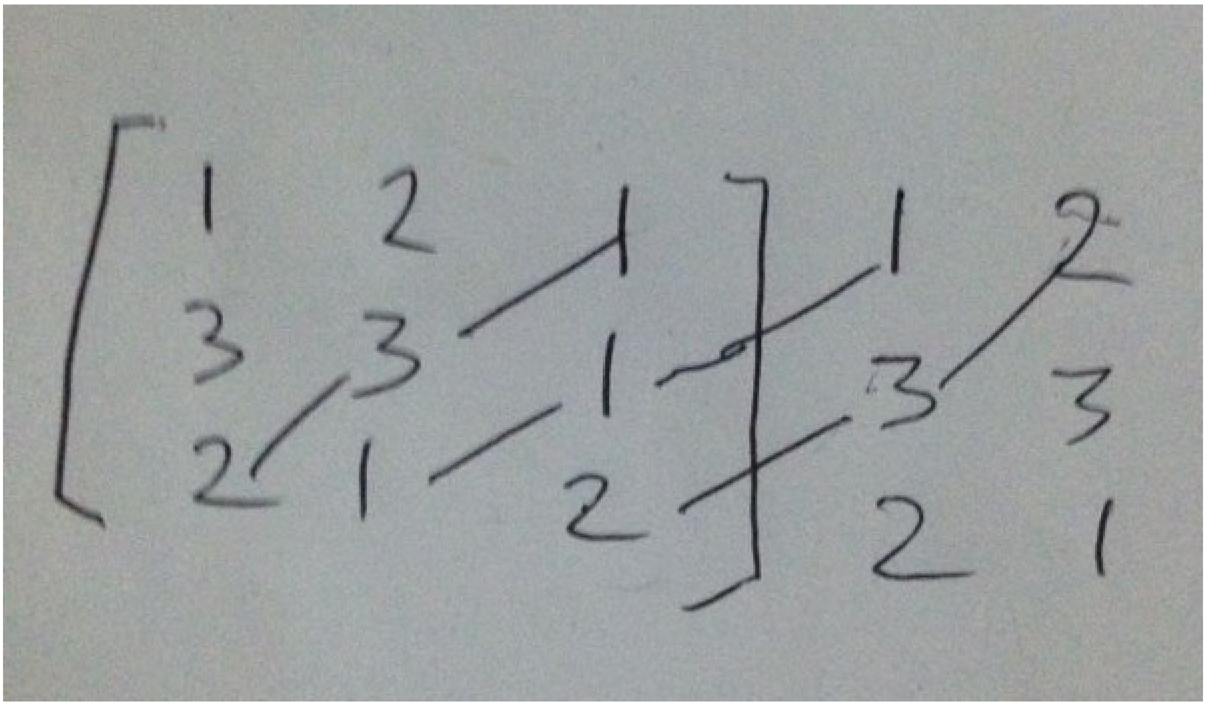
To calculate |A| of matrix A using sarrus method :



We added 2 columns from matrix beside our original matrix, so we get the sum of the products of three diagonal north-west to south-east lines of matrix elements :

(1 * 3 * 2) + (2 * 1 * 2) + (1 * 3 * 1)

Then we need to get the sum of the products of three diagonal south-west to north-east lines of elements :



(2 * 3 * 1) + (1 * 1 * 1) + (2 * 3 * 2)

Based on above informations, we got this:

|A| = ((1 * 3 * 2) + (2 * 1 * 2) + (1 * 3 * 1)) – ((2 * 3 * 1) + (1 * 1 * 1) + (2 * 3 * 2))

|A| = (6 + 4 + 3) – (6 + 1 + 12) = -6

Laplace Method

Calculating determinant of 3×3 matrix using laplace is simple.

Based on literature, to calculate 3×3 matrix using laplace :

The Laplace formula for the determinant of a 3 × 3 matrix is

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

So the matrix is splitted into 3 small matrices which 2×2 matrix, where a,b and c are constants. Those 3 small 2×2 matrices are permutation from this columns and rows:

$$\begin{vmatrix} d & e & f \\ g & h & i \end{vmatrix}$$

the 2×2 matrices :

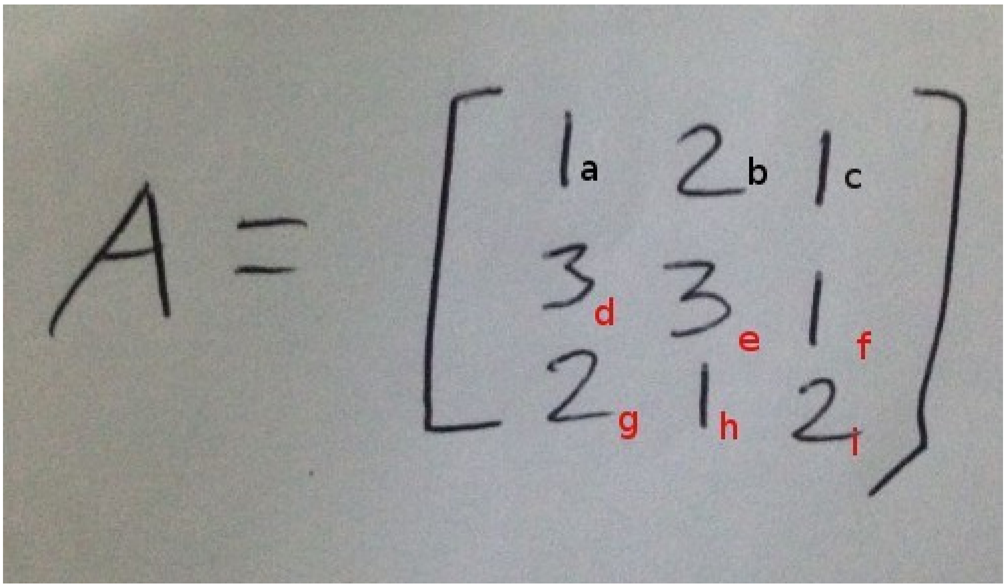
$$\begin{vmatrix} \text{[redacted]} & e & f \\ & h & i \end{vmatrix}$$

$$\begin{vmatrix} d & \text{[redacted]} & f \\ g & & i \end{vmatrix}$$

$$\begin{vmatrix} d & e & \text{[redacted]} \\ g & h & \end{vmatrix}$$

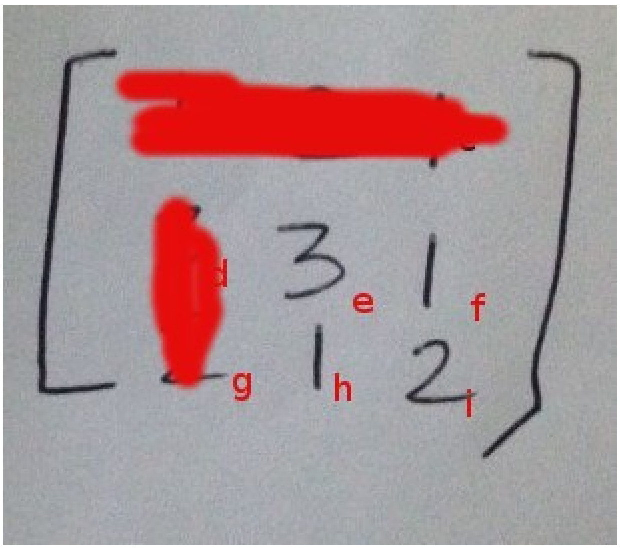
Back again to A matrix:

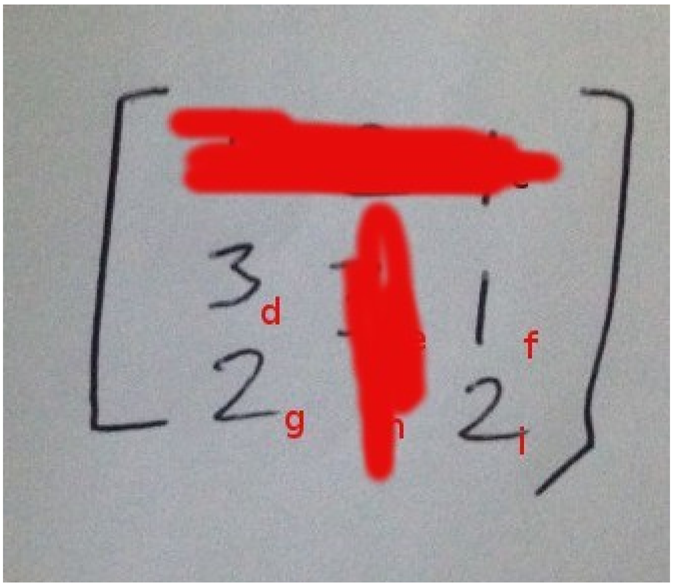
$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

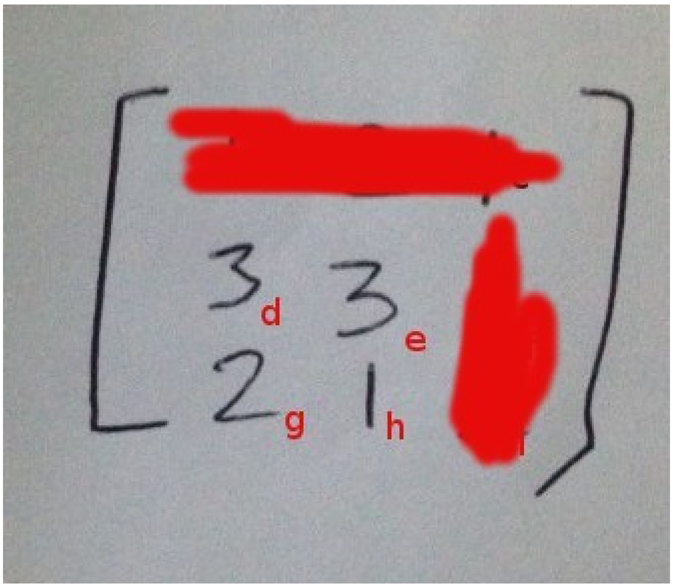


a = 1 , b = 2, c = 1

We got 2×2 permutation matrices :







$$|A| = 1\cdot\begin{vmatrix}3 & 1\\ 1 & 2\end{vmatrix} - 2\begin{vmatrix}3 & 1\\ 2 & 2\end{vmatrix} + 1\begin{vmatrix}3 & 3\\ 2 & 1\end{vmatrix}$$

determinant of first matrix = 5

determinant of second matrix = 4

determinant of third matrix = -3

so we got : (1 * 5) – (2 * 4) + (1 * -3) = 5 – 8 – 3 = -6